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Mathematics

9709/12

Paper 1 Pure Mathematics 1

October/November 2020

Question No (2)

- 2 The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression.

Solution:

sum to infinity

$$S_{\infty} = \frac{a}{1-r}$$

Given Terms of G.P ,

$$2p + 6, -2p, \quad p + 2$$

First Term , a = 2p + 6

$$\text{Common Ratio , } r = \frac{\text{second term}}{\text{First term}} = \frac{-2p}{2p + 6}$$

sum to infinity

$$S_{\infty} = \frac{a}{1-r}$$

$$\begin{aligned} &= \frac{2p + 6}{1 - \left(\frac{-2p}{2p + 6}\right)} = \frac{2p + 6}{1 + \frac{2p}{2p + 6}} \\ &= \frac{2p + 6}{\frac{2p + 6 + 2p}{2p + 6}} \end{aligned}$$

$$S_{\infty} = \frac{(2p + 6)(2p + 6)}{4p + 6} \rightarrow (1)$$

As consecutive common ratio are equal

$$\Rightarrow \frac{-2p}{2p + 6} = \frac{p + 2}{-2p}$$

$$(-2p)(-2p) = (p + 2)(2p + 6)$$

$$4p^2 = 2p^2 + 6p + 4p + 12$$

$$4p^2 - 2p^2 - 10p - 12 = 0$$

$$2p^2 - 10p - 12 = 0$$

$$p^2 - 5p - 6 = 0$$

factorize

$$p^2 - 6p + p - 6 = 0$$

$$p(p - 6) + 1(p - 6) = 0$$

$$(p - 6)(p + 1) = 0$$

$$p - 6 = 0, \quad p + 1 = 0$$

$$p = 6, -1$$

As P is +ve, So p = 6

put P = 6 in equation (1)

$$S_{\infty} = \frac{(2 \times 6 + 6)(2 \times 6 + 6)}{4(6) + 6}$$

$$S_{\infty} = \frac{(18)(18)}{30} = 10.8$$

$$\mathbf{S_{\infty} = 10.8}$$