Cambridge International AS & A Level

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Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2020**

Question No(2)

2 **(a)** Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$.

(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$.

Solution:

$(\boldsymbol{a})$

$3\cos\theta = 8\tan\theta$

$3\cos\theta = \frac{8\sin\theta}{\cos\theta}$ $\qquad \because\ \tan\theta = \frac{\sin\theta}{\cos\theta}$

$3cos^2\theta = 8\sin\theta$

$3(1 - sin^2\theta) - 8sin\theta = 0$ $\qquad \because\ cos^2\theta = 1 - sin^2\theta$

$3 - 3sin^2\theta - 8sin\theta = 0$

$-3sin^2\theta - 8\sin\theta + 3 = 0$

$-(3sin^2\theta + 8\sin\theta - 3) = 0$

$\boldsymbol{3sin^2\theta + 8\sin\theta - 3 = 0}$

(b)

$$3sin^2\theta + 8\sin\theta - 3 = 0$$

$$factorize$$

$$3sin^2\theta + 9sin\theta - \sin\theta - 3 = 0$$

$$3\sin\theta(\sin\theta + 3) - 1(\sin\theta + 3) = 0$$

$$(\sin\theta + 3)(3\sin\theta - 1) = 0$$

$\sin\theta + 3 = 0$

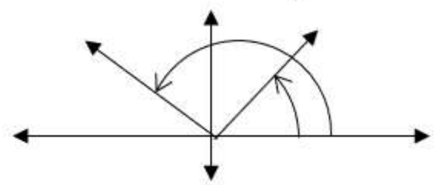
$\sin\theta = -3 \;\; neglected \quad \because -1 \le \sin\theta \le 1$

$3\sin\theta - 1 = 0$

$3\sin\theta = 1$

$\sin\theta = \dfrac{1}{3}$

$\sin\theta \;\; is + ve \; in \; I \; and \; II \; quadrant$



$Basic \; angle$

$\propto = sin^{-1}\left(\dfrac{1}{3}\right)$

$\propto = 19.5^0$

$required \; angles \; are$

$\propto, \qquad 180 - \propto$

$19.5^0, \qquad 180 - 19.5^0$

$19.5^0, \qquad 160.5^0$

$required \;\; acute \; angle \; is$

$19.5^0 \quad \because this \; is \; acute \; angle$