

Cambridge International AS & A Level

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Mathematics

9709/52

Paper 5 Probability & Statistics 1

October/November 2020

Question No (2)

- 2 A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$.

(b) Draw up the probability distribution table for X .

(c) Given that $E(X) = \frac{15}{8}$, find $\text{Var}(X)$.

Solution:

(a)

$P(\text{sadie takes exactly one red ball})$

$$= P(RBB \text{ or } BRB \text{ or } BBR)$$

$$= \left(\frac{5}{8} \times \frac{3}{7} \times \frac{2}{6}\right) + \left(\frac{3}{8} \times \frac{5}{7} \times \frac{2}{6}\right) + \left(\frac{3}{8} \times \frac{2}{7} \times \frac{5}{6}\right)$$



$$= \frac{30}{336} + \frac{30}{336} + \frac{30}{336}$$

$$= \frac{30+30+30}{336}$$

$$= \frac{90}{336}$$

$$= 0.268$$

(b)

$P(\text{sadie takes no red ball}) = P(BBB)$

$$= \left(\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}\right)$$

$$= \frac{1}{56}$$

$P(\text{sadie takes 2 red balls}) = P(RRB \text{ or } RBR \text{ or } BRR)$

$$= \left(\frac{5}{8} \times \frac{4}{7} \times \frac{2}{6}\right) + \left(\frac{5}{8} \times \frac{3}{7} \times \frac{4}{6}\right) + \left(\frac{3}{8} \times \frac{5}{7} \times \frac{4}{6}\right)$$

$$= \frac{15}{28}$$

$$p(\text{sadie takes 3 red balls}) = p(RRR)$$

$$= \left(\frac{5}{8} \times \frac{4}{7} \times \frac{3}{6}\right)$$

$$= \frac{5}{28}$$

Distribution table for X

X	0	1	2	3
$P(X=x)$	$\frac{1}{56}$	$\frac{15}{56}$	$\frac{15}{28}$	$\frac{5}{28}$

③ Given $E(X) = \frac{15}{8}$, $\text{Var}(X) = ?$

Probability Formula

$$E(X) = \sum_{i=1}^n x_i p_i$$

$$\text{Var}(X) = \sum_{i=1}^n x_i^2 p_i - (E(X))^2$$

$$\text{Var}(X) = \sum_{i=1}^n x_i^2 p_i - (E(X))^2$$

$$= \left[(0)^2 \left(\frac{1}{56} \right) + (1)^2 \left(\frac{15}{56} \right) + (2)^2 \left(\frac{15}{28} \right) + (3)^2 \left(\frac{5}{28} \right) \right] - \left(\frac{15}{8} \right)^2$$

$$= \frac{225}{56} - \frac{225}{64}$$

$$= \frac{225}{448}$$

$$= 0.502 \quad (3 \text{ sf})$$