

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

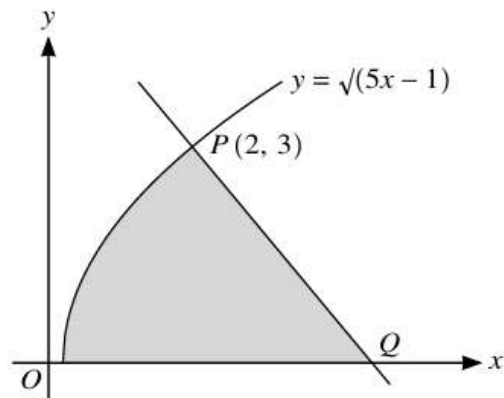
Topic 8-Integration

Question No (31)

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Question No (31)

The diagram shows part of the curve $y = \sqrt{5x - 1}$ and the normal to the curve at the point $P(2, 3)$. This normal meets the x -axis at Q .

- (i) Find the equation of the normal at P .
- (ii) Find, showing all necessary working, the area of the shaded region.

Solution

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(i) Given equation of curve

$$y = \sqrt{5x-1} \rightarrow (1)$$

$$y = (5x-1)^{1/2}$$

Differentiate w.r.t x

$$\frac{dy}{dx} = \frac{1}{2} (5x-1)^{1/2-1} \frac{d}{dx} (5x-1)$$

$$= \frac{1}{2} (5x-1)^{-1/2} (5-0)$$

$$\frac{dy}{dx} = \frac{5}{2\sqrt{5x-1}}$$

Gradient of tangent at $P(2,3)$

$$\frac{dy}{dx} = \frac{5}{2\sqrt{5(2)-1}}$$

$$\frac{dy}{dx} = \frac{5}{2\sqrt{9}} = \frac{5}{2 \times 3} = \frac{5}{6}$$

$\Rightarrow$ gradient of normal at $P = -\frac{6}{5}$

(gradient of tangent $\times$ gradient of normal $= -1$)

$\therefore$ Equation of normal at P is

$$y - y_1 = -\frac{6}{5} (x - x_1)$$

$$y - 3 = -\frac{6}{5} (x - 2)$$

$$5(y-3) = -6(x-2)$$

$$5y - 15 = -6x + 12$$

$$6x + 5y = 15 + 12$$

$$6x + 5y = 27$$

(ii)

Equation of line PQ

$$6x + 5y = 27$$

from part (i)

As this line intercepts the x-axis at Q, so

$$\text{Put } y = 0$$

$$6x + 5(0) = 27$$

$$x = \frac{27}{6}$$

$$x = 4.5$$

$$\therefore Q(4.5, 0)$$

Equation of curve

$$y = \sqrt{5x-1}$$

given

As the curve meets the x-axis, so at point

$$S, y = 0$$

$$\Rightarrow y = \sqrt{5x-1} \text{ will become}$$

$$0 = \sqrt{5x-1}$$

$$0 = 5x - 1$$

$$x = \frac{1}{5} = 0.2$$

∴ point $S(0.2, 0)$

Area of the shaded region = Area under curve PSR +
Area of $\triangle PQR$

$$= \int_{0.2}^2 y \, dx + \frac{1}{2} \text{ base} \times \text{height}$$

$$= \int_{0.2}^2 \sqrt{5x-1} \, dx + \frac{1}{2} (QR)(PR)$$

$$= \int_{0.2}^2 (5x-1)^{1/2} \, dx + \frac{1}{2} (4.5-2)(3)$$

$$= \left| \frac{(5x-1)^{3/2}}{(3/2)(5)} \right|_{0.2}^2 + 3.75$$

$$= \left| \frac{(5x-1)^{3/2}}{(3/2)(5)} \right|_{0.2}^2 + 3.75$$

$$= \frac{2}{15} \left| (5x-1)^{3/2} \right|_{0.2}^2 + 3.75$$

$$= \frac{2}{15} \left[(5(2)-1)^{3/2} - (5(0.2)-1)^{3/2} \right] + 3.75$$

$$= \frac{2}{15} \left[(9)^{3/2} - 0 \right] + 3.75 = \frac{2}{15} (27) + 3.75$$

$$= 7.35 \text{ unit}^2$$

