Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 6-Series

Question No (11)

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(a) A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector.

(b) The first, second and third terms of a geometric progression are $2k + 3$, $k + 6$ and k, respectively. Given that all the terms of the geometric progression are positive, calculate

(i) the value of the constant k,

(ii) the sum to infinity of the progression.

Solution

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As given in question statement, angles of the 6 sectors are in A.P,

$a, a+d, a+2d, a+3d, a+4d, a+5d$

By given condition

angle of largest sector = 4 times angle of smallest sector

$\Rightarrow a+5d = 4a$

$5d = 4a - a$

$5d = 3a \longrightarrow (1)$

As sum of 6 sectors angles is equal to 2π

$\Rightarrow S_6 = 2\pi \longrightarrow (2)$

Formula for the sum of nth arithmatic progression is

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_6 = \frac{6}{2}[2a + (6-1)d]$

$2\pi = 3[2a + 5d]$ $\quad \because S_6 = 2\pi$

$\frac{2\pi}{3} = 2a + 5d \longrightarrow (3)$

put value of Equation ① in ③

$$2a + 3a = \frac{2\pi}{3}$$

$$5a = \frac{2\pi}{3}$$

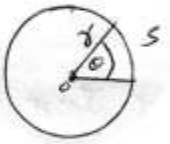
$$a = \frac{2\pi}{15}$$

∴ angle of the smallest sector, $a = \frac{2\pi}{15}$

(as a is the smallest sector and c is the largest sector in this question)

Arc length formula

$$S = r\theta$$



Arc length of smallest sector

$$S = r\theta$$

$$= 5 \times \frac{2\pi}{15}$$

$$S = \frac{2\pi}{3}$$

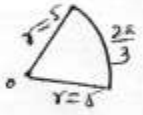
∵ $r = 5, \theta = \frac{2\pi}{15}$ given

∴ perimeter of smallest sector

$$= r + S + r$$

$$= 5 + \frac{2\pi}{3} + 5$$

$$= 12.1 \text{ cm}$$



(b)(i) As the given three terms $2K+3, K+6, K$ are in G.P.

In G.P common ratio is

$$r = \frac{\text{Second term}}{\text{First term}}$$

As in G.P common ratio between two consective terms are equal.

$$\Rightarrow \frac{K+6}{2K+3} = \frac{K}{K+6}$$

$$(K+6)^2 = K(2K+3)$$

$$K^2+12K+36 = 2K^2+3K$$

$$2K^2+3K-K^2-12K-36=0$$

$$K^2-9K-36=0$$

By factorization

$$K^2-12K+3K-36=0$$

$$K(K-12)+3(K-12)=0$$

$$(K-12)(K+3)=0$$

$K-12=0$

$K=12$

$K+3=0$

$K=-3$

ignore as K is +ve

(ii) Given 3 terms in G.P

$2K+3,\ K+6,\ K$

$2(12)+3,\ 12+6,\ 12$ $\quad \because K=12$ part(i)

$24+3,\ 18,\ 12$

$27,\ 18,\ 12$

$\Rightarrow a=27,\quad r=\frac{\text{second term}}{\text{First term}}$

$=\frac{18}{27}$

$r=\frac{2}{3}$ $\quad |r|<1$

sum to infinity formula for G.P when $|r|<1$ is

$S_\infty=\frac{a}{1-r}$

$=\frac{27}{1-\frac{2}{3}}$

$=\frac{27}{\frac{1}{3}}$

$=27\times 3$

$S_\infty=81$