

Cambridge International AS & A Level

Mathematics 9709

Paper 1 Pure Mathematics 1

Topic 2-Functions

Question No (11)

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Question No (11)

The functions f and g are defined for $x \in \mathbb{R}$ by

$$f: x \rightarrow 4x - 2x^2,$$

$$g: x \rightarrow 5x + 3.$$

(i) Find the range of f

(ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots.

Solution

$f: x \rightarrow 4x - 2x^2$
 $f(x) = 4x - 2x^2$
 and
 $g: x \rightarrow 5x + 3$
 $g(x) = 5x + 3$

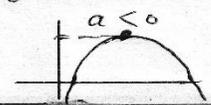
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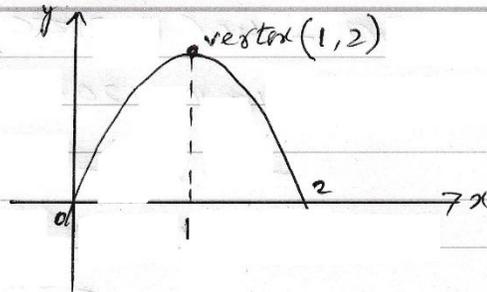
$f(x) = 4x - 2x^2$
 $f(x) = 2x(2 - x)$
 critical values
 $2x = 0$, $2 - x = 0$
 $x = 0$, $x = 2$

AS $f(x) = 4x - 2x^2$
 $= -2(x^2 - 2x)$
 $= -2(x^2 - 2(1)x)$
 $= -2(x^2 - 2(1)x + (1)^2 - (1)^2)$
 $= -2(x - 2(1)x + (1)^2) - 2x - (1)^2$
 $y = -2(x - 1)^2 + 2$

Knowledge Sharing

$\therefore y = 4x - 2x^2$
 $a = -2 < 0$ $\therefore y = ax^2 + bx + c$
 so it will have max value - parabola face up





As max point vertex is $(1, 2)$, so range of $f(x) \leq 2$ so $y \leq 2$

(ii)

As

$$f(x) = 4x - 2x^2$$

$$g(x) = 5x + 3$$

Given

$$g(f(x)) = k$$

$f(x)$ is input to $g(x)$

$$g(4x - 2x^2) = k$$

now $4x - 2x^2$ is input to $g(x)$

$$5(4x - 2x^2) + 3 = k$$

$$20x - 10x^2 + 3 = k$$

$$-10x^2 + 20x + 3 - k = 0$$

$$-(10x^2 - 20x - 3 + k) = 0$$

$$10x^2 - 20x + k - 3 = 0$$

As $g(f(x)) = k$ has equal roots, so

$$b^2 - 4ac = 0$$

discriminant
 $b^2 - 4ac$

$$\Rightarrow (-20)^2 - 4(10)(k-3) = 0$$

$$400 - 40(k-3) = 0$$

$$400 - 40k + 120 = 0$$

$$-40k = -520$$

$$k = \frac{-520}{-40}$$

$$k = 13$$