

Cambridge International AS & A Level

Mathematics

9709/12

Paper 1 Pure Mathematics 1

October/November 2024

Question No(2)

Question No (2)

The first term of an arithmetic progression is -20 and the common difference is 5.

(a) Find the sum of the first 20 terms of the progression.

.....

It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

(b) Find the value of k .

Solution:

(a)

Given data

first term, $a = -20$

common difference, $d = 5$

number of terms, $n = 20$

Sum to n terms of an A.P is

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Here

$n =$ number of terms

$a =$ first term

$d =$ common difference

$$\begin{aligned}
 S_{20} &= \frac{20}{2} [2 \times -20 + (20-1)5] \\
 &= 10 [-40 + 19(5)] \\
 &= 10 [-40 + 95] \\
 &= 10 [55]
 \end{aligned}$$

(6)

$S_{20} = 550$
 So sum of first 20 terms is 550.

(b) It is given that the sum of the first $2K$ terms is 10 times the sum of the first K terms.
 Find the value of K .

Solution According to the given condition

$$S_{2K} = 10 S_K$$

By using

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

we have

$$\frac{2K}{2} [2a + (2K-1)d] = 10 \left[\frac{K}{2} (2a + (K-1)d) \right]$$

$$K [2a - 20 + (2K-1)d] = 10 \left[\frac{K}{2} (2a - 20 + (K-1)d) \right]$$

$$K [-40 + (2K-1)d] = 5K [-40 + (K-1)d]$$

$$-40K + 10K^2 - 5K = -200K + 25K^2 - 25K$$

$$-40K + 10K^2 - 5K + 200K - 25K^2 + 25K = 0$$

$$180K - 15K^2 = 0$$

$$15K(12-K) = 0$$

$$15K = 0, 12-K = 0 \Rightarrow K = 12$$

