

Cambridge International AS & A Level

Mathematics

9709/12

Paper 1 Pure Mathematics 1

October/November 2024

Question No(2)

Question No (2)

The first term of an arithmetic progression is -20 and the common difference is 5.

- (a) Find the sum of the first 20 terms of the progression.
-

It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

- (b) Find the value of k .

Solution:

(a)

Given data

first term, $a = -20$
 common difference, $d = 5$
 number of terms, $n = 20$

Sum to n terms of an A.P is

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Here

$n =$ number of terms
 $a =$ first term
 $d =$ common difference

$$S_{20} = \frac{20}{2} [2 \times -20 + (20-1)5]$$

$$= 10 [-40 + 19(5)]$$

$$= 10 [-40 + 95]$$

$$= 10 [55]$$

$$S_{20} = 550 \quad (6)$$

So sum of first 20 terms is 550.

(b) It is given that the sum of the first $2k$ terms is 10 times the sum of the first k terms.

Find the value of k .

Solution

According to the given condition

$$S_{2k} = 10 S_k$$

By using

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

we have

$$\frac{2k}{2} [2a + (2k-1)d] = 10 \left[\frac{k}{2} (2a + (k-1)d) \right]$$

$$k [2 \times -20 + (2k-1)5] = 10 \left[\frac{k}{2} (2 \times -20 + (k-1)5) \right]$$

$$k [-40 + (2k-1)5] = 5k [-40 + (k-1)5]$$

$$-40k + 10k^2 - 5k = -200k + 25k^2 - 25k$$

$$-40k + 10k^2 - 5k + 200k - 25k^2 + 25k = 0$$

$$180k - 15k^2 = 0$$

$$15k(12-k) = 0$$

$$15k = 0, 12-k = 0 \Rightarrow k = 12$$

