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Mathematics 9709/12

Paper 1 Pure Mathematics 1 **October/November 2023**

Question No (1)

1 The coefficient of x^3 in the expansion of $(3 + 2ax)^5$ is six times the coefficient of x^2 in the expansion of $(2 + ax)^6$.

Find the value of the constant a.

Solution:

Binomial expansion

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n}b^n$$

$$(3+2ax)^5 = \binom{5}{0}(3)^5 + \binom{5}{1}(3)^4(2ax)^1 + \binom{5}{2}(3)^3(2ax)^2 + \binom{5}{3}(3)^2(2ax)^3 + \cdots + \binom{5}{5}(2ax)^5$$

The term containing x^3 in the expansion of $(3+2ax)^5$ is

$$\binom{5}{3}(3)^2(2ax)^3$$

$$\frac{5\times4\times3}{3\times2\times1} \times 2^3 a^3 x^3$$

$$10 \times 9 \times 8\, a^3 x^3$$

$$720\, a^3 x^3$$

$\therefore$ coefficient of $x^3 = 720a^3$

Similarly the term containing x^2 in the expansion of $(2+ax)^6$ is

$$\binom{6}{2}(2)^{6-2}(ax)^2$$

$$\frac{6\times5}{2\times1} \times 2^4 a^2 x^2$$

$$240\, a^2 x^2$$

$\therefore$ coefficient of $x^2 = 240a^2$

According to given condition

coefficient of $x^3 = 6 \times$ coefficient of x^2

$$720a^3 = 6(240a^2)$$

$$720a^3 = 1440a^2$$

$$720a = 1440$$

$$a = 2$$