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Mathematics 9709/12

Paper 1 Pure Mathematics 1 **May/June 2023**

Question No (2)

2 The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q. It is given that $p + q = 276$.

Find the possible values of the constant a.

Solution:

Binomial formula

$$(a+b)^n = {}^nC_0\, a^{n-0} b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \cdots + {}^nC_n\, a^{n-n} b^n$$

Applying the binomial formula for $(x+a)^6$,

$$(x+a)^6 = {}^6C_0\, x^{6-0}(a)^0 + {}^6C_1\, x^{6-1}(a)^1 + {}^6C_2\,(x)^{6-2}(a)^2 + \cdots + {}^6C_6\,(x)^{6-6}(a)^6$$

From the observation of the above expansion

${}^6C_2\,(x)^{6-2}(a)^2$ has term x^4 or coefficient .

$${}^6C_2\,(x)^{6-2}(a)^2 = \frac{6\times 5}{2\times 1}\, x^4 (a^2)$$

$$= 15a^2 x^4$$

$\Rightarrow p = 15a^2$ (as coefficient of x^4 is p)

Applying binomial formula for $(ax+3)^4$, we have

$$(ax+3)^4 = {}^4C_0\,(ax)^{4-0}(3)^0 + {}^4C_1\,(ax)^{4-1}(3)^1 + {}^4C_2\,(ax)^{4-2}(3)^2 + \cdots + {}^4C_4\,(ax)^{4-4}(3)^4$$

From above expansion term containing x^2, coefficient

$^4C_2 (ax)^{4-2} (3)^2 = \frac{4\times3}{2\times1} (ax)^2 (9)$

$= 54a^2x^2$

$\Rightarrow q = 54a^2$ (as question condition coefficient of x^2 is q)

As $p+q = 276$ (given)

$\Rightarrow 15a^2 + 54a^2 = 276$ (By putting values of p and q)

$69a^2 = 276$

$a^2 = 276/69 = 4$

$a^2 = 4$

$a = \pm 2$