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Mathematics 9709/12

Paper 1 Pure Mathematics 1 **October/November 2022**

Question No (1)

1 Points A and B have coordinates $(5, 2)$ and $(10, -1)$ respectively.

(a) Find the equation of the perpendicular bisector of AB.

(b) Find the equation of the circle with centre A which passes through B.

Solution:

(a)

$A(5,2)$, $B(10,-1)$

> Mid-point formula for two points
>
> $A(x_1, y_1)$, $B(x_2, y_2)$
>
> $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$

Mid-point of the line joining AB =

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$= \left(\frac{5+10}{2}, \frac{2-1}{2} \right)$$

$$= \left(\frac{15}{2}, \frac{1}{2} \right)$$

Gradient of line $AB = \frac{y_2-y_1}{x_2-x_1}$

$$= \frac{-1-2}{10-5}$$

$$= -3/5$$

$\Rightarrow$ gradient of perpendicular bisector of $AB = 5/3$ $\quad \because m_1 \times m_2 = -1$

Equation of perpendicular bisector

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{2} = \frac{5}{3}\left(x - \frac{15}{2}\right)$$

$$y - \frac{1}{2} = \frac{5x}{3} - \frac{75}{6}$$

$$\frac{2y-1}{2} = \frac{10x-75}{6}$$

$$2y - 1 = 2\left(\frac{10x-75}{6}\right)$$

$$2y - 1 = \frac{10x-75}{3}$$

$$3(2y-1) = 10x - 75$$

$$6y - 3 = 10x - 75$$

$$6y = 10x - 75 + 3$$

$$6y = 10x - 72$$

$$6y = 2(5x - 36)$$

$$3y = 5x - 36$$

(b) radius of circle, $r = |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(10-5)^2 + (-1-2)^2}$$

$$= \sqrt{(5)^2 + (-3)^2}$$

$$r = \sqrt{25+9}$$

$$r = \sqrt{34}$$

∴ Equation of circle

$$(x-5)^2 + (y-2)^2 = (\sqrt{34})^2$$

$$x^2 - 10x + 25 + y^2 - 4y + 4 = 34$$

Equation of circle with centre (h,k) and radius r:

$$(x-h)^2 + (y-k)^2 = r^2$$

$$x^2 + y^2 - 10x - 4y + 29 = 34$$

$$x^2 + y^2 - 10x - 4y - 5 = 0$$